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PHYSICS IN DAILY LIFE:

# BRAVE DUCKS

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Remember how hard it was to first break the sound barrier? It took several fatal attempts by brave pilots before Charles ('Chuck') Yeager finally managed to fly faster than the speed of sound on the 14th of October, 1947. The problem was: by the time an aircraft approaches the speed of sound, the sound wave crests pile up in front of the plane. It then has to push through this barrier of compressed air in order to go faster than the waves. Once it is faster than the sound waves, an interesting situation occurs, quite similar to the case of a bullet moving at supersonic speed. The wave fronts produced have an enveloping circular cone, the 'Mach cone'. It is easy to see that the half apex angle of the cone, θ, is related to the speed of sound $c$ and the speed of the plane $v$ by $\sin\theta = c/v$. Since there are no sound waves outside the Mach cone, the plane will pass us before we actually hear its sound.

Sound waves bear many analogies to water waves. Look at a duck, for example, speeding through a deep pond. See the V-shaped pattern of waves trailing the swimming duck? Doesn't it look like he is fighting the 'wave barrier' of water in front of him and producing a two-dimensional version of the Mach cone? Brave duck!

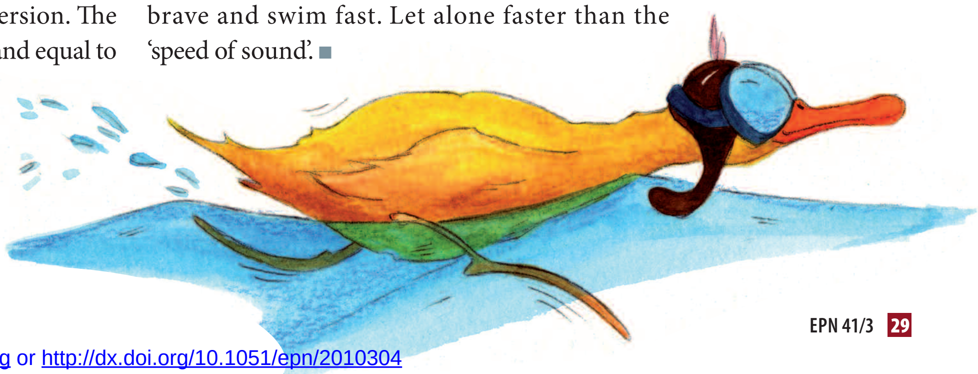
This certainly is an appealing thought. But it's wrong. What we may perceive as a 2-D version of a 'Mach cone' actually consists of two envelopes of a feathered pattern of dispersive waves.

Despite the analogies between water waves and sound waves, there are a few essential differences. Sound waves in air travel at a fixed speed without dispersion. The phase velocity $c$ is equal for all wavelengths and equal to the group velocity. For supersonic flight this leads to the simple expression for the 'Mach angle' given above.

Water waves are much more complicated. They travel at the interface of two media, and are governed by gravity. Let us look at the deep-water limit, which is a good approximation for the duck as well as for ships on the ocean. Unlike sound waves in air, the phase velocity of the waves $V$ depends on the wavelength, with long waves traveling faster than short waves. They follow the dispersion law $V = \sqrt{(g/k)}$ where $g$ is the acceleration of gravity and $k$ the wave number $2\pi/\lambda$. In other words, the speed of the waves is proportional to the square root of their wave length. At any speed of the duck or the ship, there will be waves running along with the same speed, whereas in the supersonic-flight case all waves are overtaken by the plane.

The complicated behaviour of the waves behind a duck or a ship in deep water was first worked out by Lord Kelvin (William Thomson), and is often referred to as 'Kelvin wake pattern' or 'Kelvin ship waves'. Kelvin was the first to find that, indeed, the wave pattern is bounded at either side by a straight line at an angle of 19,5 degrees with respect to the direction of the ship. This sounds like an awkward angle, resulting from a rather lengthy derivation. The angle may sound less awkward if we write it down in its precise form, as arcsin (1/3). In turn, the 1/3 results from the fact that the phase velocity given above is twice the group velocity. But the important thing is: this odd angle is fixed and characteristic for this type of wave. It has nothing to do with speed.

Too bad for the duck: In order to produce the V-shaped Kelvin wave pattern, he doesn't have to be brave and swim fast. Let alone faster than the 'speed of sound'. ■





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